Received: *September 21, 2000*, Accepted: *October 17, 2000*

Brane potentials and moduli spaces

George Papadopoulos* **and David Tong**

Department of Mathematics, Kings College London
Strand, London WC2R 2LS, UK
E-mail: gpapas@mth.kcl.ac.uk, tong@mth.kcl.ac.uk

Abstract: It is shown that the supergravity moduli spaces of D1-D5 and D2-D6 brane systems coincide with those of the Coulomb branches of the associated non-abelian gauge theories. We further discuss situations in which worldvolume brane actions include a potential term generated by probing certain supergravity backgrounds. We find that in many cases, the appearance of the potential is due to the application of the Scherk-Schwarz mechanism. We give some examples and discuss the existence of novel supersymmetric brane configurations.

Keywords: p-branes, Brane Dynamics in Gauge Theories.

*Theory Division, CERN, 1211, Geneva 23, Switzerland

JHEP10(2000)032

Contents

1. Introduction **1**

2. The D1-D5 brane system **3**
- 2.1 Supergravity 3
- 2.2 Gauge theory 4

3. The D2-D6 brane system **6**
- 3.1 Supergravity 6
- 3.2 Gauge theory 7

4. Probe brane potentials **9**
- 4.1 SS mechanism for transverse scalars 9
- 4.2 Non-marginal BPS backgrounds and potentials 12
- 4.3 SS mechanism for Born-Infeld fields 13
- 4.4 Potentials in gauge theory 17

1. Introduction

Recently, there has been much progress in understanding the moduli spaces of multiple black holes in the supergravity context [1]–[6]. For earlier work in this area, see [8, 9, 10]. Given the emphasis of string theory over the past few years, it is natural to ask whether one can successfully compare such moduli spaces with those of gauge theories. In the first part of this paper we point out that, in two simple cases, the moduli spaces do indeed coincide with the quantum corrected Coulomb branch of a corresponding non-abelian Yang-Mills theory.

While similar in spirit to the pre-AdS/CFT probe calculations [11, 12], the scenario considered here involves an important difference: rather than restricting attention to test-particles (branes) moving in a fixed background, the background is determined by the positions of other branes which are themselves self-gravitating and dynamical. Therefore we are necessarily dealing with the scattering of multiple branes. Multiple D3-branes have been treated as probes in the past [13, 14], but only in situations in which various simplifications ensure that the probes do not interact with each other, and the resulting moduli space is simply the symmetric product of the single probe moduli space. In the present case we find that interactions between branes do occur, but are restricted to two-body forces. Specifically, we show that the supergravity moduli spaces of D2-D6 and D1-D5 brane configurations, each of which preserve eight supercharges, coincide with the Coulomb branches of $d = 3$, $\mathcal{N} = 4$, and $d = 2$, $\mathcal{N} = (4, 4)$, non-abelian Yang-Mills theory, respectively. Of course, the supergravity and gauge theory calculations have different ranges of validity and agreement between them points to the existence of a non-renormalisation theorem which, in the present case, reduces to a strong constraint on the geometry of the manifold due to the preservation of eight supercharges and the remaining Lorentz invariance: the moduli metric is restricted to be hyperKähler (HK) for the D2-D6-brane configuration and strong hyperKähler with torsion (HKT) for D1-D5-brane configuration. As is the norm in such calculations, the classical supergravity result is reproduced by one-loop effects in the gauge theory. We argue that there are no non-perturbative corrections.

In the second half of this paper, we discuss situations in which the low-energy dynamics of branes includes a potential term. Such potentials may be generated in the context of compactifications by using the Scherk-Schwarz (SS) mechanism [15]. Since most worldvolume actions of various branes can be related by Kaluza-Klein type of compactifications, the SS mechanism can be used to generate potentials on the brane.[1] In particular one begins from the standard Dirac-Born-Infeld type of action of a (D-, M-, NS-) p-brane and after giving an appropriate expectation value to either a *transverse scalar* or to a *Born-Infeld* (BI) type of field or to both, one finds after compactification in n-directions a $(p - n)$-brane action which has a scalar potential term. In many cases, the scalar potential is just a constant shift in the reduced action but if the p-brane is placed in an appropriate supergravity background, then a non-trivial potential can appear. The scalar potentials that appear by placing D-branes in a constant B-field or in the non-trivial compactification of the M2-brane in [16] are examples of this. There are many cases that one can consider by choosing different supergravity backgrounds and by placing various brane probes in them. However we shall not explore all these possibilities here. Instead we shall present some new examples including compactification in the presence of a constant B-field and non-trivial compactification of D-brane worldvolume actions in the presence of a ten-dimensional KK-monopole.

For the cases associated with non-trivial compactifications in a KK-monopole background, we shall show that there is an alternative bulk explanation for the presence of a potential in the $(p - n)$-brane action. In particular we shall find that the same potential appears on a $(p - n)$-brane probe placed in a non-marginal BPS supergravity background. Such backgrounds were first found in [20] and further explored in [21, 22].

[1]In the context of the supergravity approach to branes this has been used in [17] and further explored in [18]. The appearance of potentials in M-theory compactifications with non-trivial background form field strengths have been investigated in [19].

In addition we shall investigate the presence of supersymmetric solutions in the various probe actions with a scalar potential and examine their properties. Some of these have the interpretation of rotating branes. Finally, we also discuss the generation of potentials in the context of supersymmetric gauge theories.

In the section two we investigate the D1-D5 system, and in section three we examine the D2-D6 system. Finally in section four we discuss brane potentials.

2. The D1-D5 brane system

2.1 Supergravity

Our starting point is the much studied D1-D5 system, comprised of Q_1 D-strings lying in the 01 directions and Q_2 D5-branes lying in the 012345 directions. The metric of the associated supergravity solution is,

$$ds^2 = H_1^{-1/2} H_2^{-1/2} ds^2(\mathbb{R}^{(1,1)}) + H_1^{1/2} H_2^{-1/2} ds^2(\mathbb{T}^4) + H_1^{1/2} H_2^{1/2} ds^2(\mathbb{R}^4)\,, \tag{2.1}$$

where the directions 2345 are compactified on $\mathbb{T}^4$, and the harmonic functions, H_1 and H_2, associated with the D-strings and D5-branes, respectively, are given by

$$H_I = h_I + \sum_{A=1}^{N_I} \frac{\lambda_A^I}{|\vec{x} - \vec{x}^{IA}|^2}\,. \tag{2.2}$$

From the form of these functions, we see that the D-strings have been organised into N_1 clusters, each consisting of λ_A^1, $A = 1, \ldots, N_1$ branes, with position in the transverse $\mathbb{R}^4$ given by $\vec{x}^{1A}$. Similarly, the D5-branes have been split into N_2 clusters, consisting of λ_A^2, $A = 1, \ldots, N_2$ branes with position $\vec{x}^{2A}$. Clearly $Q_I = \sum_{A=1}^{N_I} \lambda_A^I$. Notice further that the asymptotic volume V of the torus $\mathbb{T}^4$ as $|x| \to \infty$ is given by,

$$V = \sqrt{\det(h_1^{1/2} h_2^{-1/2}\,\delta_{ab})} \longrightarrow \frac{h_1}{h_2}\,,$$

where $a, b = 2, \ldots, 5$ label the coordinates of the torus. Upon dimensional reduction on $\mathbb{T}^4$ to six dimensions, this supergravity solution becomes a string. We are interested in the moduli space of such solitons, with the $4(N_1 + N_2)$ positions $\vec{x}^{IA}$ considered as collective coordinates. The low-energy dynamics of these objects is then described by a two-dimensional sigma model with (4,4) supersymmetry whose target space is the moduli space. The computation of the moduli metric can be done by adapting similar results for black holes given in [5]. We find,

$$ds^2_{BH} = \sum_A \left(h_2 \lambda_A^1 |d\vec{x}^{1A}|^2 + h_1 \lambda_A^2 |d\vec{x}^{2A}|^2\right) + \sum_{A,B} \lambda_A^1 \lambda_B^2 \frac{|d\vec{x}^{1A} - d\vec{x}^{2B}|^2}{|\vec{x}^{1A} - \vec{x}^{2B}|^2}\,. \tag{2.3}$$

This metric is compatible with two-dimensional (4,4)-supersymmetry if it is supplemented with an appropriate torsion term which in turn induces a Wess-Zumino term in the effective theory; the torsion three-form is closed. The moduli space is a strong HKT manifold associated with two hypercomplex structures induced from those on $\mathbb{R}^4$ corresponding to a basis of self-dual and anti-self-dual two-forms. Note that the metric (2.3) displays interaction terms only between branes of different type. There are no two-derivative forces between branes of the same type, reflecting the fact that in isolation each species of brane preserves 16 supersymmetries. Such interactions would appear at fourth order in derivatives and it remains a challenge to derive them through supergravity methods. Further note that in the case that the D-strings are on top of the D5-branes, $\vec{x}^{1A} = \vec{x}^{2A}$, and $\lambda^1_A = \lambda^2_A$, the metric on the moduli space becomes that of Shiraishi [10] (see [5] for $\lambda^1_A \neq \lambda^2_A$).

2.2 Gauge theory

We turn now to the gauge theory of the D1-D5 system. While attention is usually focussed upon the Higgs branch of this theory, we will here be interested in the Coulomb branch, parametrising the motion of the D1- and D5-branes in the overall transverse space $\mathbb{R}^4$. The gauge theory in question resides on the $1+1$-dimensional intersection of the D1- and D5-branes, has $\mathcal{N} = (4,4)$ supersymmetry (eight supercharges) and gauge group $\mathrm{U}(Q_1) \times \mathrm{U}(Q_2)$ with coupling constants e_I. The ratio of the coupling constants is determined by the volume of the torus,

$$\frac{e_1^2}{e_2^2} = V = \frac{h_1}{h_2}\,. \tag{2.4}$$

The matter coupling consists of an adjoint hypermultiplet for each gauge group, together with a single hypermultiplet in the bi-fundamental. Let us focus on the bosonic matter content of the above multiplets. For each of the vector multiplets, this consists of a two-dimensional gauge field, together with four real adjoint scalars which we will denote $\vec{\phi}^I$, where the index $I = 1, 2$ labels the two gauge groups. Each hypermultiplet consists of a further four real scalars. The vector multiplets and adjoint hypermultiplets arise from strings with both ends on the D-string or both ends on the D5-branes, and furnish a representation of $\mathcal{N} = (8,8)$ supersymmetry. This is reduced to $\mathcal{N} = (4,4)$ by strings stretched between the D5 and D1-branes, giving rise to the bi-fundamental hypermultiplet. While vacuum moduli spaces do not exist in two dimensions, progress can still be made by deriving a low-energy sigma-model description of the gauge theory in the spirit of a Born-Oppenheimer approximation. The target space is then referred to as the vacuum moduli space. Our theory has two branches of vacua: an $8(Q_1 + Q_2)$-dimensional Coulomb branch parametrised by the scalars in the two vector multiplets and two adjoint hypermultiplets, and a $4Q_1Q_5$-dimensional Higgs branch parametrised by the scalars in the adjoint and bi-fundamental hypermultiplets. The latter is relevant when the D-strings are absorbed as instantons inside the D5-brane. For the present purpose, it is the Coulomb branch that is of interest. The vacuum expectation values of the scalars in the vector multiplets parametrise the positions of the D-branes in 6789 directions, while those of the scalars in the adjoint hypermultiplets determine the positions of the D-strings and Wilson lines of the D5-branes in the 2345 directions. Upon dimensional reduction to six dimensions, we will be interested only in the 6789 positions: the supergravity calculation does not capture the modes specified by the adjoint hypermultiplet scalars. Thus we set the vacuum expectation values of these scalars to zero and concentrate on the $4(Q_1+Q_2)$-dimensional sub-manifold of the Coulomb branch parametrised by four adjoint scalars $\vec{\phi}^I$.

The usual commutator terms in the scalar potential ensure that $\vec{\phi}^I$ are simultaneously diagonalisable. To compare to the supergravity result, we further restrict attention to the $4(N_1+N_2)$-dimensional subspace of the Coulomb branch, on which $\vec{\phi}^I = \mathrm{diag}\,(\vec{\phi}^{IA})$, $A=1,\ldots,N_I$, where each entry $\vec{\phi}^{IA}$ is proportional to the $(\lambda^I_A \times \lambda^I_A)$ unit matrix. This results in the gauge symmetry breaking, $\mathrm{U}(Q_I) \to \prod_{A=1}^{N_I} \mathrm{U}(\lambda^I_A)$. The existence of surviving non-abelian gauge symmetries implies that this sub-manifold lies within a singularity of the full Coulomb branch, reflecting the presence of these extra massless excitations. Nonetheless, we may concentrate only on the subset of deformations which preserve the form of the vacuum expectation value and derive a low-energy effective action for these modes.

Classically, this sub-manifold of the Coulomb branch is described by the flat metric $(\mathbb{R}^{4N_1}/S_{N_1}) \times (\mathbb{R}^{4N_2}/S_{N_2})$, where the quotient arises from the Weyl group of the gauge theory. The singularities correspond to situations where the groups of D-strings or D5-branes become coincident and further non-abelian symmetry restoration occurs.

The metric receives one-loop corrections from integrating out massive matter, including W-bosons, off-diagonal terms of the adjoint hypermultiplet, and the bi-fundamental hypermultiplets. Importantly, the contribution from the first two of these cancel. This is obvious as together they make a $(8,8)$-supersymmetric gauge multiplet and the moduli space metric of any gauge theory with sixteen supercharges is constrained to be flat. This reflects the fact that the D1-branes and D5-branes do not interact at the two-derivative level with branes of the same type. Thus the only corrections come from the bi-fundamental hypermultiplets. Under the symmetry breaking, $\mathrm{U}(Q_I) \to \prod_{A=1}^{N_I} \mathrm{U}(\lambda^I_A)$, these decompose into $N_1 N_2$ hypermultiplets, each transforming in the bi-fundamental representation of a single pair $\mathrm{U}(\lambda^1_A) \times \mathrm{U}(\lambda^2_B)$, $A=1,\ldots,N_1$ and $B=1,\ldots,N_2$, and with mass $|\vec{\phi}^{1A} - \vec{\phi}^{2B}|$. The one-loop corrected Coulomb branch metric is given by [14, 23],

$$ds^2_{\mathrm{gauge}} = \sum_{A=1}^{N_1} \frac{\lambda^1_A}{e_1^2}\, |d\vec{\phi}^{1A}|^2 + \sum_{B=1}^{N_2} \frac{\lambda^2_A}{e_2^2}\, |d\vec{\phi}^{2A}|^2 + \sum_{A=1}^{N_1}\sum_{B=1}^{N_2} \lambda^1_A \lambda^2_B \frac{|d\vec{\phi}^{1A} - d\vec{\phi}^{2B}|^2}{|\vec{\phi}^{1A} - \vec{\phi}^{2B}|^2}\,, \tag{2.5}$$

where the factors of λ in the first two, classical, terms come from tracing over block-diagonal matrices, and the third term arises from integrating out the bi-fundamental hypermultiplets. In particular, the $1/|\text{mass}|^2$ behaviour is typical of one-loop corrections in two-dimensional gauge theories as may be seen by simple dimensional analysis. Notice that in the simplest case in which the position of only a single D-string is allowed to vary in the presence of fixed D5-branes, (2.5) reduces to the five-brane metric [14, 23]. The generalization of this result is that the full Coulomb branch metric (2.5) coincides with the metric on the moduli space of the D1-D5 system (2.3) if we identify $1/e_I^2 = |\epsilon_{IJ}|h_J$. This further ensures that equation (2.4) is satisfied.

As commented above, supersymmetry requires that the metric be accompanied by a suitable torsion term. Such terms are indeed generated at one loop in the gauge theory [23] and that the resulting low-energy dynamics is given by a two-dimensional $(4,4)$-supersymmetric sigma-model.

We have shown that the classical moduli space metric of five-dimensional black holes coincides with the one-loop corrected Coulomb branch of an associated two-dimensional gauge theory. For gauge groups of rank one, it can be argued that the restrictions of HKT, together with Spin(4) symmetry inherited from the R-symmetry of the gauge theory, require that the metric receives no further corrections. While we know of no such analysis for higher rank gauge groups, it seems plausible that similar behaviour occurs. In particular, on the Coulomb branch in two dimensions, there are no candidate instanton solutions to give semi-classical non-perturbative corrections.

3. The D2-D6 brane system

3.1 Supergravity

The investigation of the D2-D6 brane configuration is similar to that of the D1-D5 system of the previous section. Indeed, the two configurations are related by T-duality. The D2-D6 brane supergravity solution that we shall consider is

$$ds^2 = H_1^{-1/2} H_2^{-1/2} ds^2(\mathbb{R}^{(1,2)}) + H_1^{1/2} H_2^{-1/2} ds^2(\mathbb{T}^4) + H_1^{1/2} H_2^{1/2} ds^2(\mathbb{R}^3)\,, \qquad (3.1)$$

where

$$H_I = h_I + \sum_{A=1}^{N_I} \frac{\lambda_{IA}}{|\vec{x} - \vec{x}^{IA}|}\,, \qquad (3.2)$$

for $I = 1, 2$ are now harmonic functions on $\mathbb{R}^3$ associated with D2- and D6-branes, respectively. The moduli space that we shall examine is that parametrised by the positions $\vec{x}^{IA}$, $I = 1, 2$, in the overall transverse three-space of the D2- and D6-branes.

Upon reduction on $\mathbb{T}^4$, we are left with a membrane type of solution in six dimensions and the effective theory is a three-dimensional sigma model with eight supersymmetry charges. Supersymmetry requires that the sigma model target space is a HK manifold. The moduli metric restricted on the positions $\vec{x}^{IA}$, $I = 1, 2$, can be computed by appropriately adapting the results on black hole moduli spaces in [5]. It was found that the moduli metric is

$$ds^2_{BH} = \sum_A \left(h_2\lambda^1_A |d\vec{x}^{1A}|^2 + h_1\lambda^2_A |d\vec{x}^{2A}|^2 \right) + \sum_{A,B} \lambda^1_A \lambda^2_B \frac{|d\vec{x}^{1A} - d\vec{x}^{2B}|^2}{|\vec{x}^{1A} - \vec{x}^{2B}|} . \tag{3.3}$$

Note that, unlike for the D1-D5-brane configuration, we have not identified all collective coordinates of the D2-D6 system. Indeed, the moduli space of positions has dimension $3(N_1 + N_2)$ while the moduli space of the system is expected to have $4(N_1 + N_2)$ dimensions because it must be HK. The absence of these collective coordinates arises in the calculation of the moduli metric because perturbations of high rank gauge potentials along the worldvolume of the D2-brane were ignored. Such perturbations vanish in the black hole case but they do not for the D2-D6 brane configuration. However as we shall review below, the moduli metric (3.3) admits a unique hyperKähler completion by addition of $(N_1 + N_2)$ periodic coordinates.

3.2 Gauge theory

An analysis similar to that of the previous section may be given for the gauge theory, which consists of a three-dimensional $\mathcal{N} = 4$ (eight supercharges) gauge multiplet with gauge group $\mathrm{U}(Q_1)\times\mathrm{U}(Q_2)$, a hypermultiplet in the adjoint representation of the gauge group and a further hypermultiplet in the bi-fundamental. While the bosonic matter content of the hypermultiplet is unchanged in different dimensions, the vector multiplet now contains a three-dimensional gauge field and only three real, adjoint scalars which we again denote as $\vec{\phi}^I$. Once again, when combined with the adjoint hypermultiplets, these fields fill out a representation of the $\mathcal{N} = 8$ supersymmetry algebra and this is broken to $\mathcal{N} = 4$ only by the presence of the bi-fundamental hypermultiplet. Dimensional reduction of this theory to two-dimensions results in the model discussed in the previous section.

Once again, the three-dimensional gauge group is broken as $\mathrm{U}(Q_I) \to \prod_{A=1}^{N_I} \mathrm{U}(\lambda^I_A)$ and we restrict ourselves to the relevant subspace of the Coulomb branch which is now of dimension $3(N_1 + N_2)$. The gauge theory supplies us with the remaining $(N_1 + N_2)$ periodic scalars, σ^I_A, courtesy of the dual photons that are released upon breaking the gauge group.

As in the two-dimensional case, the one-loop corrections from the adjoint hypermultiplets cancel those from the W-boson multiplets, and only the bi-fundamental hypermultiplets contribute, making it simple to immediately write down the one-loop corrected metric on the Coulomb branch [24], which is of the Lee-Weinberg-Yi type [25] (see also [26]),

$$ds^2 = g_{AIBJ} d\vec{\phi}^{AI} \cdot d\vec{\phi}^{BJ} + (g^{-1})^{AIBJ} \psi_{AI} \psi_{BJ} , \tag{3.4}$$

where the first term is given by,

$$\sum_{A=1}^{N_1} \frac{\lambda_A^1}{e_1^2} |d\vec{\phi}^{1A}|^2 + \sum_{B=1}^{N_2} \frac{\lambda_B^2}{e_2^2} |d\vec{\phi}^{2B}|^2 + \sum_{A=1}^{N_1}\sum_{B=1}^{N_2} \lambda_A^1 \lambda_B^2 \frac{|d\vec{\phi}^{1A} - d\vec{\phi}^{2B}|^2}{|\vec{\phi}^{1A} - \vec{\phi}^{2B}|} \tag{3.5}$$

and is seen to reproduce the supergravity result (3.3). Notice that, in contrast to (2.5) the one-loop correction in three dimensions has $1/|\mathrm{mass}|$ behaviour. The second term in (3.4) is the hyperKähler completion mentioned above, with

$$\psi_{AI} = d\sigma_{AI} + \vec{\omega}_{AIBJ} d\vec{\phi}^{BJ}\,, \tag{3.6}$$

where $\vec{\omega}$ is defined by $\nabla\times\vec{\omega} = \nabla g$. The manifold has (N_1+N_2) tri-holomorphic isometries which act on the periodic coordinates σ_{AI} by constant shifts. Such a manifold is known as toric HK. These symmetries are preserved within perturbation theory and the strong restriction of toric hyperKählarity thereby ensures that there are no higher loop corrections to the metric. However, instanton effects break this symmetry, and one may worry about their presence. In three dimensions, the relevant semiclassical configurations are monopoles. Importantly, in $\mathcal{N}=4$ three-dimensional gauge theories, and in contrast to their four-dimensional cousins, one-loop effects around the background of the instanton do not cancel [27]. Moreover, in gauge groups of rank $r \geq 2$, when the Coulomb branch is interpreted in terms of soliton scattering these terms give rise to r-body interactions [28]. As mentioned in the introduction, such interactions do not appear from the supergravity perspective. To see that such terms do not appear in the gauge theory either, one must determine whether instantons do indeed contribute to the metric. In fact it is simpler to examine the four-fermi term which is included in the supersymmetric completion of the metric. Instantons can contribute to such a term only if they have precisely four fermionic zero modes and no more. Thus, in order to determine whether instantons contribute in the present case, we need only count fermionic zero modes. The necessary observation is that the vector multiplet and adjoint hypermultiplet form an $\mathcal{N}=8$ multiplet which, for fundamental instantons, has 8 zero modes; too many to contribute to two derivative terms. One may wonder if four of these can be lifted through couplings to the bi-fundamental hypermultiplets. However, these hypermultiplets do not couple directly to the adjoint hypermultiplet, and no such term can arise. Similar comments apply to the multi-instanton case. Therefore instantons do not contribute to two derivative terms in these theories, and the one-loop result (3.5) is exact.

We conclude this section with the remark that the moduli metric of the D2-D6 brane system is T-dual to the moduli metric of the D1-D5 brane system under Buscher type duality. This can be seen by adapting the results of [1] to this case. It appears that the type-II T-duality that relates the D2-D6 and D1-D5 brane systems induces the Buscher T-duality on their moduli spaces.

4. Probe brane potentials

The action of a brane probe placed in a supergravity background that preserves some supersymmetry is invariant, after gauge fixing kappa-symmetry and world-volume reparametrisations, under as many supersymmetry transformations as those preserved by the background. However whether or not supersymmetry is preserved in certain phases of the theory depends upon the existence of a supersymmetric ground configuration. In many well-studied examples, including those of the previous sections, the derivative expansion of the probe action starts with velocity dependent terms. In such a situation, a supersymmetric configuration is achieved by simply ensuring that the probe is stationary and appropriately oriented with respect to the background. However there are many cases for which the derivative expansion starts with a potential term. In many cases this potential arises due to the presence of a non-vanishing expectation value for one or more fields of a p-brane action compactified to a $(p-n)$-brane action. The $(p-n)$-brane action then develops a potential with coefficient that depends on the above expectation values. This is the SS mechanism. The preservation of as many as eight supercharges in the probe action does not necessarily rule out such a potential [29, 30]. The issue then is whether or not a supersymmetric configuration can be found which can be interpreted as the supersymmetric vacuum of the theory.

4.1 SS mechanism for transverse scalars

We shall present two examples of brane action compactifications with the SS-mechanism applied to one of the transverse scalars. These involve the M2- and M5-branes in a KK-monopole background. The former example has been already considered in [16] but here we shall investigate the supersymmetric ground configuration for the standard Taub/NUT metric.

For this we begin with the M-theory solution of a KK-monopole extended in 0123456(10). The corresponding supergravity solution is

$$ds^2 = ds^2(\mathbb{R}^{(1,6)}) + H^{-1}(d\theta+\omega)^2 + H ds^2(\mathbb{R}^3)\,, \tag{4.1}$$

where $H = 1+p/|y|$ is the harmonic function on $\mathbb{R}^3$, $dH = {}^*d\omega$ and $y \in \mathbb{R}^3$. The non-flat part of the metric is the familiar Taub-NUT hyper-Kähler metric; the eleventh coordinate has been identified with $\theta = x^{10}$. It is known that such solution preserves 1/2 of the bulk supersymmetry with supersymmetry projection

$$\Gamma_7\Gamma_8\Gamma_9\Gamma_\theta\epsilon = \epsilon\,. \tag{4.2}$$

In this background we place a M2-brane probe and use static gauge in the directions 012. Next we compactify the direction 2 on S^1 in such a way that we keep only the zero modes for all field in all directions apart from that for the transverse scalar θ for which we set

$$\partial_2 \theta = q \,, \tag{4.3}$$

where q is a constant. This KK-ansatz is consistent and it is a special case of that proposed in [15]. For compactifications of the M2-brane see [16]. After integrating over S^1, the effective action for such a system in the small derivative approximation is

$$S = \frac{1}{2} \int d^2x \, \Big(\delta_{ab} \eta^{\mu\nu} \partial_\mu z^a \partial_\nu z^b + H \delta_{ij} \eta^{\mu\nu} \partial_\mu y^i \partial_\nu y^j + \\ + H^{-1} \eta^{\mu\nu} (\partial_\mu \theta + \omega_i \partial_\mu y^i)(\partial_\nu \theta + \omega_j \partial_\nu y^j) + q^2 H^{-1} \Big) \,, \tag{4.4}$$

where $\{z^a; a = 1, \ldots, 4\}$ are the transverse scalars in directions 3456, $\{\theta, y^i; i = 1, 2, 3\}$ are the three transverse scalars associated with the KK-monopole (directions (10)789) and $\mu, \nu = 0, 1$. This action, apart from the standard kinetic term, also contains a potential which is the length of the tri-holomorphic vector field of the Taub-NUT geometry. The lower-dimensional lagrangian describes a string propagating is a KK-background in the presence of a potential. As we shall see, there is an alternative interpretation of the action (4.4) as describing a string propagating in the background of a non-marginal ten-dimensional KK-monopole/ D6-brane background.

This system possesses a unique supersymmetric ground state in which a planar string lies at the origin of the KK-monopole, $y^i = 0$, so the potential vanishes, and the rest of the transverse scalars are constant. The supergravity solution associated with the Taub/NUT metric preserves 1/2 of the bulk supersymmetry apart from near the origin of the KK-monopole where all bulk supersymmetry is preserved. The string planar worldvolume solution preserves 1/2 of that of the background and so 1/4 of the bulk.

Next let us examine the theory near the origin of the KK-monopole. The Taub/NUT metric near the origin is flat. In the natural flat coordinates (w^1, w^2, w^3, w^4), we have the relation $|y| = |w|^2$. In this case, the action (4.4) reduces to that of a free theory with scalar potential

$$V = \frac{q^2}{p} |w|^2 \,, \tag{4.5}$$

i.e. the fields along the KK-monopole directions are massive. The supersymmetry preserved by the planar string solution is 1/2 of the bulk.

Apart from the string solution above, there exists another supersymmetric solution to the classical equations of motion given by,

$$z^a = \text{const} \,, \qquad y^i = \text{const} \neq 0 \,, \qquad \theta = -qt \,. \tag{4.6}$$

The solution (4.6) describes a string which rotates with constant angular velocity $-q$ in the θ direction. Observe that the solution is not invariant under the string worldvolume Lorentz transformations. The solution (4.6) can be easily lifted to a solution for the M2-brane as follows:

$$z^a = \text{const}\,, \qquad y^i = \text{const}\,, \qquad \theta = -qt + qx^2\,, \tag{4.7}$$

describing a M2-brane with the x^2 direction wrapped on θ with winding number q, so $q \in \mathbb{Z}$, and rotating around θ with angular velocity $-q$.

To investigate the number of supersymmetry charges preserved by the string solution (4.6), it is enough to investigate the number of supersymmetry charges preserved by the lifted M2-brane solution (4.7). For this, we can use the supersymmetry condition associated with the kappa-symmetry supersymmetry projection [31, 32, 33]. A brief calculation reveals that the supersymmetry projections required are

$$\Gamma_0\Gamma_2\epsilon = \epsilon\,, \qquad \Gamma_0\Gamma_1\Gamma_2\epsilon = \epsilon\,. \tag{4.8}$$

Therefore the solution (4.6) preserves 1/4 of supersymmetry of the background or 1/8 of the bulk as an immediate consequence of (4.2).

For our next example, we keep the same KK-monopole background as above, but replace the M2-brane probe with an M5-brane probe [34] in the directions 012345. We use static gauge and set the three-form self-dual field of the M5-brane equal to zero because it does not contribute to the potential in what follows. Next we compactify the M5-brane in the direction x^5 on S^1 and keep only the zero modes for all transverse fields apart from the transverse scalar θ for which we set

$$\partial_5\theta = q\,, \tag{4.9}$$

where q is a constant. In the small velocity approximation, the effective action for such a system after integrating over S^1 is

$$\begin{aligned} S = \frac{1}{2}\int d^5x\, \Big(&\eta^{\mu\nu}\partial_\mu z\partial_\nu z + H\delta_{ij}\eta^{\mu\nu}\partial_\mu y^i\partial_\nu y^j + \\ &+ H^{-1}\eta^{\mu\nu}(\partial_\mu\theta + \omega_i\partial_\mu y^i)(\partial_\nu\theta + \omega_j\partial_\nu y^j) + q^2H^{-1}\Big)\,, \end{aligned} \tag{4.10}$$

where z is the transverse scalar in the x^6 direction, $\{(\theta, y^i); i = 1,2,3\}$ are transverse scalars along the KK-monopole and $\mu,\nu = 0,1,\ldots,4$. The action (4.10) describes a D4-brane in a KK-monopole background which apart from the standard kinetic term for the transverse scalars also contains a potential as in the string case above.

The analysis is now similar to the M2-brane probe. There exists a unique supersymmetric ground state in which the D4-brane lies at $y^i = 0$ with all other transverse scalars constant. There further exists a classical solution of the D4-brane lagrangian (4.10) given by,

$$z = \text{const}\,, \qquad y^i = \text{const} \neq 0\,, \qquad \theta = -qt\,, \tag{4.11}$$

which can be lifted as a M5-brane solution as

$$z = \text{const}\,, \qquad y^i = \text{const}\,, \qquad \theta = -qt + qx^5\,. \tag{4.12}$$

The solution (4.11) describes a D4-brane which rotates with constant angular velocity $-q$ in the θ direction while the M5-brane wraps and rotates in the same direction. Again both the D4- and M5-brane solutions are not Lorentz invariant under the appropriate worldvolume Lorentz transformations. The supersymmetry projections associated with the M5-brane solution above are

$$\Gamma_0\Gamma_5\epsilon = \epsilon\,, \qquad \Gamma_0\Gamma_1\ldots\Gamma_5\epsilon = \epsilon\,. \tag{4.13}$$

Therefore using (4.2) we find that the solution preserves 1/8 of the bulk supersymmetry.

The above is clearly a special case of a more general class of constructions where an M-brane is placed in a supergravity background with a killing isometry. Then using T- and S-dualities, one can construct D- and NS-brane actions with non-trivial potentials. For example one can construct actions with potentials for all Dp-branes in a KK-monopole background by compactifying or T-dualizing the D4-brane action above.

It is also straightforward consider the case where the original background involves many KK-monopoles by allowing the harmonic function H to have many centres. In such a case the relevant action will also be given by (4.10) but now it will depend on the new harmonic function. In this case apart of the solution that we have considered there are other Q-kink type of solutions that preserve some supersymmetry; for work in this direction see [16, 37].

4.2 Non-marginal BPS backgrounds and potentials

There is an alternative interpretation for the action (4.4) as describing a fundamental string propagating in a ten-dimensional KK-monopole/D6-brane background. To illustrate this, we take the eleven-dimensional KK-monopole background (4.1) and change coordinates as

$$\sigma = qz + \theta\,, \qquad \rho = z\,, \tag{4.14}$$

where z is one of the coordinates in $\mathbb{R}^{(1,6)}$ and q is identified with the parameter in the SS mechanism for the M2-brane. Then we reduce the solution to ten-dimensions along ρ. The resulting ten-dimensional solution is

$$\begin{aligned} ds^2 &= (1+q^2H^{-1})^{1/2}\left[ds^2(\mathbb{R}^{(1,5)}) + \frac{1}{H+q^2}(d\sigma+\omega)^2 + H ds^2(\mathbb{R}^3)\right],\\ e^{\frac{4}{3}\phi} &= (1+q^2H^{-1})\,, \qquad A_1 = -H^{-1}(1+q^2H^{-1})^{-1}q(d\sigma+\omega)\,. \end{aligned} \tag{4.15}$$

In this background, we place a fundamental string and choose a static gauge along a two-dimensional subspace of $\mathbb{R}^{(1,5)}$. Now the effective lagrangian of such a fundamental string in the *small derivative* and *small* q approximation coincides with that of (4.4) after relabeling $\sigma = \theta$. In this approximation q is in the same order as the derivatives of the transverse scalars.

The above computation can be adapted easily for the case of interpreting the result of the SS mechanism for the M5-brane. In particular, the action (4.10) describes the dynamics of a D4-brane probe in the background (4.15) in the small derivative and small q approximation.

In the case of D-branes a similar interpretation for the actions with potential can be given. However in this case the non-marginal background that it is probed is T-dual to the one that we have started with. For later use, the shall consider the SS reduction of the D3-brane in the background of a ten-dimensional KK-monopole. Changing coordinates as above and T-dualizing along the direction ρ. The T-dual background is

$$\begin{aligned} ds^2 &= ds^2(\mathbb{R}^{(1,4)}) + (1+q^2H^{-1})^{-1}d\rho^2 + \frac{1}{H+q^2}(d\sigma+\omega)^2 + H ds^2(\mathbb{R}^3)\,, \\ e^{2\phi} &= (1+q^2H^{-1})^{-1}\,, \qquad H_3 = -d\Big((1+q^2H^{-1})^{-1}d\rho\wedge(qd\sigma+\omega)\Big)\,, \end{aligned} \tag{4.16}$$

which describes a non-marginal ten-dimensional KK-monopole/NS5-brane bound state. Probing this background with a D2-brane, the dynamics of the D2-brane is described in the small derivative and small q approximation by the action

$$\begin{aligned} S = \frac{1}{2}\int d^3x\ \Big(&\eta^{\mu\nu}\delta_{ab}\partial_\mu z^a\partial_\nu z^b + \eta^{\mu\nu}\partial_\mu\rho\partial_\nu\rho + H\delta_{ij}\eta^{\mu\nu}\partial_\mu y^i\partial_\nu y^j + \\ &+ H^{-1}\eta^{\mu\nu}(\partial_\mu\sigma+\omega_i\partial_\mu y^i)(\partial_\nu\sigma+\omega_j\partial_\nu y^j) + q^2H^{-1}\Big)\,, \end{aligned} \tag{4.17}$$

where $\{z^a; a,b = 1,2\}$ are the scalars along $\mathbb{R}^{(1,4)}$ transverse to the worldvolume directions of the D2-brane. Clearly the FI parameter associated with the potential is determined by the expectation value of a transverse scalar.

4.3 SS mechanism for Born-Infeld fields

An alternative way to find brane actions that exhibit a scalar potential is to give an expectation value to a BI type of field. This in particular can be applied in the case of D-branes and for that of M5-branes. For the latter case see also [38].

Here we shall investigate the case involving D6-brane probes in the D2-D6 brane system. The supergravity solution for the D2-D6 brane system is

$$\begin{aligned} ds^2 &= H^{-1}ds^2(\mathbb{R}^{(1,2)}) + ds^2(\mathbb{R}^4) + Hds^2(\mathbb{R}^3)\,, \qquad e^{\phi} = H^{1/2}\,, \\ A_7 &= d\,\mathrm{vol}(\mathbb{R}^{(1,2)}\oplus\mathbb{R}^4)(H^{-1}-1)\,, \qquad A_3 = d\,\mathrm{vol}(\mathbb{R}^{(1,2)})(H^{-1}-1)\,, \end{aligned} \tag{4.18}$$

where ϕ is the dilaton, A_3 and A_7 are the R⊗R gauge potentials associated with the D2-brane and the D6-brane, respectively. We have also identify the harmonic function of the D6-brane with that of the D2-brane. For later use, the projections on the killing spinor associated with the above background are

$$\Gamma_0\Gamma_1\ldots\Gamma_6\epsilon = -\epsilon\,, \qquad \Gamma_0\Gamma_1\Gamma_2\epsilon = \epsilon\,. \tag{4.19}$$

In this background we place a D6-brane probe along the $\mathbb{R}^{(1,2)} \oplus \mathbb{R}^4$ directions by choosing the static gauge. The action of the probe is the standard Dirac-Born-Infeld (DBI) one including Chern-Simons terms. In the small derivative approximation the DBI part of the action is

$$\begin{aligned} S_{\rm BI} = \int d^7x\, \Big\{ & -H^{-1} + 1 + \frac{1}{2}H\delta_{ij}\eta^{\mu\nu}\partial_\mu y^i\partial_\nu y^j + \frac{1}{2}\delta_{ij}\delta^{ab}\partial_a y^i\partial_b y^j + \\ & + \frac{1}{4}\left[H^{-1}F_{\mu\nu}F^{\mu\nu} + 2F_{\mu a}F^{\mu a} + H^{-1}F_{ab}F^{ab}\right]\Big\}\,, \end{aligned} \tag{4.20}$$

where $y = y(x,z)$ are the three transverse scalars, $F_{\mu\nu}$ and F_{ab} is the Born-Infeld (BI) field in the directions 012 and $(a,b = 3456)$, respectively and $F_{\mu a}$ are again the components of the BI field in the mixed directions; indices are raised and lowered with respect to the flat metric. The contribution from the Chern-Simons term is

$$S_{\rm CS} = \int d^7x\, \Big\{H^{-1} - 1 + \frac{1}{4}(H^{-1}-1)F_{ab}{}^*F^{ab}\Big\}\,, \tag{4.21}$$

where the Hodge duality operation is with respect to the flat metric on $\mathbb{R}^4$. The two terms in the Chern-Simons contributions come from the D6-brane and D2-brane gauge potentials, respectively. Combining both terms we find

$$\begin{aligned} S = \frac{1}{2}\int d^7x\, \Big\{ & H\delta_{ij}\eta^{\mu\nu}\partial_\mu y^i\partial_\nu y^j + \delta_{ij}\delta^{ab}\partial_a y^i\partial_b y^j + \frac{1}{2}H^{-1}F_{\mu\nu}F^{\mu\nu} + F_{\mu a}F^{\mu a} - \\ & - \frac{1}{2}F_{ab}{}^*F^{ab} + \frac{1}{2}H^{-1}[F_{ab}F^{ab} + F_{ab}{}^*F^{ab}]\Big\}\,. \end{aligned} \tag{4.22}$$

The term involving the combination $(H^{-1}-1)$ cancels between the BI and CS terms of the action because of the BPS condition of the probe relative to the background [39].

There are several ways to compactify the above action along the directions 3456 on $\mathbb{T}^4$. For example one can perform a standard $\mathbb{T}^4$ torus compactification. The resulting action will be that of a D2-brane propagating in the background[2] Alternatively, one can perform a non-trivial compactification by allowing

$$F_{ab} = B_{ab}\,, \tag{4.23}$$

[2]Strictly speaking the compactification should be followed by a T-duality on the background. However (4.18) is invariant under T-duality along all directions on $\mathbb{T}^4$.

where B is a constant field. In such case the resulting action is

$$S = \frac{1}{2}\int d^3x \, \Big\{ H\delta_{ij}\eta^{\mu\nu}\partial_\mu y^i \partial_\nu y^j + \delta_{ij}\delta^{ab}\partial_a y^i \partial_b y^j + \frac{1}{2}H^{-1}F_{\mu\nu}F^{\mu\nu} + \\ + \delta_{ab}\eta^{\mu\nu}\partial_\mu z^a \partial_\nu z^b - \frac{1}{2}B_{ab}{}^*B^{ab} + \frac{1}{2}H^{-1}[B_{ab}B^{ab} + B_{ab}{}^*B^{ab}] \Big\}, \tag{4.24}$$

where $\{z^a\}$ are the Kaluza-Klein scalars associated with the BI field. This action again described a D2-brane in the background (4.18) but also exhibits a scalar potential with coefficient dependent on the non-vanishing expectation value of the BI field. The potential term vanishes if B is chosen to be anti-self-dual. The above compactification followed by an appropriate truncation is consistent, i.e. solutions of the reduced action are also solutions of the higher dimensional one.

Let us now discuss the supersymmetric configurations of this probe brane action. Firstly let us suppose that $B = B^-$ is anti-self-dual. In such a case the potential vanishes. A solution of the system is that of standard planar D2-brane located at a point y^i = const and z^a = const in the background with $F_{\mu\nu} = 0$. Such solution preserves 1/4 of the supersymmetry. A non-vanishing value of B does affect the number of supersymmetries preserved by the configuration. This can be easily seen by lifting this solution to that of a planar D6-brane probe and then use the supersymmetry projector arising from kappa-symmetry [32, 33]. The supersymmetry projector associated with $B = B^-$ is the same as that of the D2-brane of the background. One may view the effect of B as inducing more D2-brane charge on the original D6-brane. These new D2-branes lie parallel to those of the background and so no more supersymmetry is broken.

For another supersymmetric configuration, we decompose $B = B^+ + B^-$ into self-dual and anti-self-dual parts, we require that $B^+ \neq 0$. Moreover we seek a solution for which z^a = const and $A_\mu = A_\mu(x)$ and $y^i = y^i(x)$. Substituting these into the remaining field equations, we find

$$\partial_\mu(H\partial^\mu y^i) - \frac{1}{2}\partial^i H^{-1}(B^+_{ab}B^{+ab}) = 0\,, \qquad \partial_\mu(H^{-1}F^{\mu\nu}) = 0\,. \tag{4.25}$$

A solution for this system is

$$y^i = 0\,, \qquad F_{\mu\nu} = \text{const}\,. \tag{4.26}$$

This is the most general vacuum configuration in this sector. The investigation of supersymmetry in more subtle. The background we are considering (4.18) does not have a well defined near horizon geometry as $|y| \to 0$. Consequently, the killing spinors are not well defined at that point. However since for a generic point the background preserves 1/4 of supersymmetry, the effective theory preserves eight supersymmetry charges by continuity one may argue that the same number of supersymmetry charges survives at $y = 0$. Assuming this, we take $B^+_{34} = B^+_{56} \neq 0$, $F_{12} \neq 0$ non-zero and with the rest of the components to vanish. Lifting the D2-brane solution to that of the D6-brane probe, the naive supersymmetry conditions arising from kappa-symmetry become after using the projections (4.2) the following:

$$\frac{1}{2}\mathcal{F}_{MN}\Gamma^M\Gamma^N\epsilon = \Big[F_{12}\Gamma^1\Gamma^2 + B^+_{34}(\Gamma^3\Gamma^4+\Gamma^5\Gamma^6)\Big]\epsilon = 0\,. \tag{4.27}$$

This can be rewritten using again (4.2) as

$$F_{12}\Gamma^1\Gamma^2 + 2B^+_{34}\Gamma^3\Gamma^4 = 0\,. \tag{4.28}$$

Observe that the B^- part does not contribute in the supersymmetry condition above. This leads to a supersymmetry projection provided

$$F_{12} = \pm 2B^+_{34}\,. \tag{4.29}$$

Therefore the configuration preserves 1/2 of that of the background, and 1/8 of the bulk. Let us discuss the possible interpretation of this bound state in the bulk. This configuration clearly involves D2- and D6-branes. However, one may also view B^+ as inducing anti-D2-brane charges arising from B^+. Finally F_{12} induces, using standard arguments (see e.g. [35, 36]) after further compactifying on $\mathbb{T}^2$, D0-brane charges on the probe. Therefore the bulk configuration should have the interpretation of a D0-D2-$\bar{\text{D}}$2-D6 bound state. However, the existence of a BPS solution of the effective theory does not necessarily imply the existence of a bound state in the full string theory, as one also expects tachyonic modes to be present in the system which have not been taken into account in the above analysis [40].

A similar analysis can be done for the D1-D5 brane system leading to similar conclusion but now involving a bound state of a D-instanton, a D-string, a anti-D-string and a D5-brane. The relevant action in this case is as in (4.24) but there are some differences. One difference is that the harmonic function H which appears in the action is that on $\mathbb{R}^4$ instead of $\mathbb{R}^3$ and another is that the integration in the same action is over a string worldvolume. In addition the D1-D5 background considered here has near horizon geometry $AdS_3\times S^3\times\mathbb{R}^4$ preserving 1/2 of the bulk supersymmetry [42]. Note also that in order to introduce D-instantons one has to consider the euclidean DBI action.

It is also straightforward consider the case where the original background involves many D2-D6 branes by allowing the harmonic function H to have many centres. In such a case the relevant action will also be given by (4.22) but now the harmonic function will have many centres. In this case apart of the solution that we have considered there are other Q-kink type of solutions that preserve some supersymmetry. It would be of interest to investigate these solution further; see also [16, 37]. The above analysis can also be carried out with the full non-linear DBI action.

JHEP10(2000)032

4.4 Potentials in gauge theory

In this final section, we discuss the generation of potentials on the moduli spaces of gauge theories. Specifically, we will return to the D2-D6 system of section 2. It will suffice to consider a single D2- and N D6-branes. We take the volume of the torus $\mathbb{T}^4$ to be infinite which ensures $e_2 \to \infty$ and the dynamics of the D6-branes are frozen. The moduli space metric (3.4) reduces to the Coulomb branch of the D2-brane worldvolume theory which is given by the multi-centred Taub-NUT space,

$$ds^2 = H d\vec{\phi} \cdot d\vec{\phi} + H^{-1}(d\sigma + \vec{\omega} \cdot d\vec{\phi})^2 \tag{4.30}$$

with $H = 1/e_1^2 + N/|\vec{\phi}|$, where for $N > 1$, the metric is singular. This reflects the fact that the D2-brane is a probe in the multi-Kaluza-Klein monopole geometry of the D6-branes. In particular, the dual photon σ may be identified with the eleventh dimension. For $N > 1$, the gauge theory has a Higgs branch emanating from the origin of the Coulomb branch.

Now let us consider how things change when we introduce a Fayet-Iliopoulos (FI) parameter, ζ. Classically these terms ensure that the Coulomb branch of the gauge theory no longer exists. However, one may nevertheless derive a description of the low-energy dynamics of the vector multiplet as a massive sigma model with target space (4.30) and a potential energy U. From the classical lagrangian, the potential on the Coulomb branch is given by $\frac{1}{2}e_1^2\zeta^2$. However, in the full theory the coupling constant e_1 is replaced by its quantum corrected value, resulting in

$$U = \frac{1}{2} H^{-1} \zeta^2 \, . \tag{4.31}$$

This potential is familiar from the preceding sections; it has has a minimum at $\phi = 0$, implying an induced attractive force between the D2- and D6-branes. Note that the resulting dynamics preserves eight supercharges as can be seen by noting that the potential is proportional to the length of a tri-holomorphic Killing vector associated with the isometry $\sigma \to \sigma + c$ [29, 30].

It has been argued that the bulk interpretation of the FI parameter is as a self-dual background NS$\otimes$NS B-field [41, 40]. Indeed, open string calculations reveal an attractive force between the D2- and D6-branes in the presence of a B-field. The main evidence for the specific identification of the self-dual part of the B-field with the FI parameter comes from looking not at the Coulomb branch as above, but at the Higgs branch. For theories with several D6-branes, the Higgs branch is deformed by the FI parameter into the moduli space of a single non-commutative instanton in U(N) gauge theory. This agrees with the string theory picture of the D2-brane dissolving as an instanton in the D6-branes which, in the presence of a background B-field, support a non-commutative Yang-Mills theory.

Although the field theory has a unique supersymmetric vacua, as in the previous sections, there are further classical supersymmetric solutions, given by,

$$\vec{\phi} = \vec{\phi}_0 = \text{const.} \neq 0\,, \qquad \dot{\sigma} = \zeta \tag{4.32}$$

which may be simply seen to be a BPS solution by completing the square in the hamiltonian and noting that the residual cross-term is the Noether charge, Q, associated with the isometry that shifts σ. The energy of this state is thus $E = H^{-1}\zeta^2 = Q$, and is seen to be correlated with the separation of the D2-brane from the D6-brane.

As mentioned previously, the interpretation of these states in the full IIA string theory is unclear due to issues associated with tachyons. However, in this case, there does indeed exist a natural interpretation. One may dualise σ into the three-dimensional field strength F which, for the above solution, gives,

$$F_{12} = H^{-1}\dot{\sigma} = \zeta H^{-1}\,. \tag{4.33}$$

Alternatively, one may consider this to be a non-marginal D0-D2 bound state. This state therefore has the interpretation of a D0-D2-D6 bound state in the background of a constant NS⊗NS B-field. It preserves 1/8 of the supersymmetry of the bulk.

Acknowledgments

We would like to thank M. Aganagic, S. Das, J. Gauntlett, A. Hanany, A. Karch, J. Minahan and W. Taylor for useful discussions. G.P. would like to thank CERN for a research associateship and CALTECH for hospitality while this work was in progress. D.T. would like to thank the Center for Theoretical Physics, MIT, Boston and S. Mukhi and the Department of Theoretical Physics, TIFR, Mumbai for hospitality while this work was completed. G.P. is supported by a University Research Fellowship from the Royal Society. D.T. is supported by an EPSRC fellowship. Both authors acknowledge SPG grant PPA/G/S/1998/00613 for further support. This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E) under cooperative research agreement # DF-FCO2-94ER40818.

References

[1] G.W. Gibbons, G. Papadopoulos and K.S. Stelle, *HKT and OKT geometries on soliton black hole moduli spaces*, *Nucl. Phys.* **B 508** (1997) 623 [hep-th/9706207].

[2] J. Michelson and A. Strominger, *Superconformal multi-black hole quantum mechanics*, *J. High Energy Phys.* **09** (1999) 005 [hep-th/9908044];
R. Britto-Pacumio, J. Michelson, A. Strominger and A. Volovich, *Lectures on superconformal quantum mechanics and multi-black hole moduli spaces*, hep-th/9911066.

[3] J. Gutowski and G. Papadopoulos, *The dynamics of very special black holes*, *Phys. Lett.* **B 472** (2000) 45 [hep-th/9910022]; *The dynamics of very special black holes*, *Phys. Lett.* **B 472** (2000) 45 [hep-th/9910022].

[4] A. Maloney, M. Spradlin and A. Strominger, *Superconformal multi-black hole moduli spaces in four dimensions*, hep-th/9911001.

[5] J. Gutowski and G. Papadopoulos, *Moduli spaces for four-dimensional and five-dimensional black holes*, *Phys. Rev.* **D 62** (2000) 064023 [hep-th/0002242].

[6] D. Marolf and S. Surya, *On the moduli space of the localised 1-5 system*, *J. High Energy Phys.* **08** (2000) 018 [hep-th/0005146].

[7] Y. Degura and K. Shiraishi, *Effective field theory of slowly moving 'extreme black holes'*, *Class. and Quant. Grav.* **7** (2000) 4031 [hep-th/0006015].

[8] G.W. Gibbons and P.J. Ruback, *The motion of extreme Reissner-Nordström black holes in the low velocity limit*, *Phys. Rev. Lett.* **57** (1986) 1492.

[9] R.C. Ferrell and D.M. Eardley, *Slow motion scattering and coalescence of maximally charged black holes*, *Phys. Rev. Lett.* **59** (1987) 1617.

[10] K. Shiraishi, *Moduli space metric for maximally-charged dilaton black holes*, *Nucl. Phys.* **B 402** (1993) 399.

[11] M. Douglas, J. Polchinski and A. Strominger, *Probing five-dimensional black holes with D-branes*, *J. High Energy Phys.* **12** (1997) 003 [hep-th/9703031].

[12] J.M. Maldacena, *Branes probing black holes*, *Nucl. Phys.* **68** *(Proc. Suppl.)* (1998) 17 [hep-th/9709099].

[13] O. Aharony, J. Sonnenschein, S. Yankielowicz and S. Theisen, *Field theory questions for string theory answers*, *Nucl. Phys.* **B 493** (1997) 177 [hep-th/9611222].

[14] M.R. Douglas, D.A. Lowe and J.H. Schwarz, *Probing F-theory with multiple branes*, *Phys. Lett.* **B 394** (1997) 297 [hep-th/9612062].

[15] J. Scherk and J.H. Schwarz, *How to get masses from extra dimensions*, *Nucl. Phys.* **B 153** (1979) 61.

[16] E. Bergshoeff and P.K. Townsend, *Solitons on the supermembrane*, *J. High Energy Phys.* **05** (1999) 021 [hep-th/9904020].

[17] E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, *Duality of type II 7-branes and 8-branes*, *Nucl. Phys.* **B 470** (1996) 113 [hep-th/9601150].

[18] I.V. Lavrinenko, H. Lu and C.N. Pope, *Fibre bundles and generalised dimensional reductions*, *Class. and Quant. Grav.* **15** (1998) 2239 [hep-th/9710243].

[19] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [hep-th/9906070].

[20] J.M. Izquierdo, N.D. Lambert, G. Papadopoulos and P.K. Townsend, *Dyonic membranes*, *Nucl. Phys.* **B 460** (1996) 560 [hep-th/9508177].

[21] J.G. Russo and A.A. Tseytlin, *Waves, boosted branes and BPS states in M-theory*, *Nucl. Phys.* **B 490** (1997) 121 [hep-th/9611047].

[22] M. Costa and G. Papadopoulos, *Superstring dualities and p-brane bound states*, *Nucl. Phys.* **B 510** (1998) 217 [hep-th/9612204].

[23] D-E Diaconescu and N. Seiberg, *The Coulomb branch of* $(4,4)$*-supersymmetric field theories in two dimensions*, *J. High Energy Phys.* **07** (1997) 001 [hep-th/9707158].

[24] K. Intriligator and N. Seiberg, *Mirror symmetry in three dimensional gauge theories*, *Phys. Lett.* **B 387** (1996) 513 [hep-th/9607207].

[25] K. Lee, E. Weinberg and P. Yi, *The moduli space of many BPS monopoles for arbitrary gauge groups*, *Phys. Rev.* **D 54** (1996) 1633 [hep-th/9602167].

[26] G.W. Gibbons and P. Rychenkova, *Hyper Kähler quotient construction of BPS monopole moduli spaces*, hep-th/9608085.

[27] N. Dorey, V.V. Khoze, M.P. Mattis, D. Tong and S. Vandoren, *Instantons, three-dimensional gauge theory and the Atiyah-Hitchin manifold*, *Nucl. Phys.* **B 502** (1997) 59 [hep-th/9703228].

[28] C. Fraser and D. Tong, *Instantons, three dimensional gauge theories and monopole moduli spaces*, *Phys. Rev.* **D 58** (1998) 085001 [hep-th/9710098].

[29] L. Alvarez-Gaumé and D. Freedman, *Potentials for the supersymmetric nonlinear sigma model*, *Comm. Math. Phys.* **91** (1983) 87.

[30] G. Papadopoulos and P.K. Townsend, *Massive* (p,q)*-supersymmetric sigma models revisited*, *Class. and Quant. Grav.* **11** (1994) 2163 [hep-th/9406015].

[31] K. Becker, M. Becker and A. Strominger, *Five-branes, membranes and nonperturbative string theory*, *Nucl. Phys.* **B 456** (1995) 130 [hep-th/9507158].

[32] E. Bergshoeff, R. Kallosh, T. Ortin and G. Papadopoulos, κ*-symmetry, supersymmetry and intersecting branes*, *Nucl. Phys.* **B 502** (1997) 149 [hep-th/9705040].

[33] M. Marino, R. Minasian, G. Moore and A. Strominger, *Nonlinear instantons from supersymmetric p-branes*, *J. High Energy Phys.* **01** (2000) 005 [hep-th/9911206].

[34] P.S. Howe, E. Sezgin and P.C. West, *Covariant field equations of the M-theory five-brane*, *Phys. Lett.* **B 399** (1997) 49 [hep-th/9702008].

[35] M.R. Douglas, *Branes within branes*, hep-th/9512077.

[36] J. Polchinski, *Tasi lectures on D-branes*, `hep-th/9611050`.

[37] N.D. Lambert and D. Tong, *Kinky D-strings*, *Nucl. Phys.* **B 569** (2000) 606 [`hep-th/9907098`].

[38] S. Chakravarty, K. Dasgupta, O.J. Ganor and G. Rajesh, *Pinned branes and new non Lorentz invariant theories*, *Nucl. Phys.* **B 587** (2000) 228 [`hep-th/0002175`].

[39] A.A. Tseytlin, *'No-force' condition and BPS combinations of p-branes in 11 and 10 dimensions*, *Nucl. Phys.* **B 487** (1997) 141 [`hep-th/9609212`].

[40] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [`hep-th/9908142`].

[41] O. Aharony, M. Berkooz and N. Seiberg, *Light-cone description of (2,0) superconformal theories in six dimensions*, *Adv. Theor. Math. Phys.* **2** (1998) 119 [`hep-th/9712117`].

[42] H.J. Boonstra, B. Peeters and K. Skenderis, *Duality and asymptotic geometries*, *Phys. Lett.* **B 411** (1997) 59 [`hep-th/9706192`].

JHEP10(2000)032